10th Class 2019

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define pure quadratic equation. Give an example.

Ans An equation which contains the square of unknown variable, but no higher power is called pure quadratic equation.

Example:

$x^2 - 16 = 0$ and $4x^2 = 7$ are the pure quadratic equation.

(ii) Solve by factorization: $5x^2 = 30x$.

Ans Given, $5x^2 = 30x$

It can be written as:

$$5x^2 - 30x = 0$$

which is factorized as

$$5x(x - 6) = 0$$

Either $5x = 0$ or $x - 6 = 0$

$\Rightarrow$ $x = 0$ or $x = 6$

$\therefore$ $x = 0, 6$ are the roots of given equation. Thus, the solution set is $\{0, 6\}$.

(iii) Find the discriminant of the following equation:

$$2x^2 - 7x + 1 = 0$$

Ans Given, $2x^2 - 7x + 1 = 0$

Here, $a = 2$, $b = -7$, $c = 1$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (-7)^2 - 4(2)(1) \\ &= 49 - 8 = 41 \end{aligned}$$

(iv) Write the quadratic equation having roots $-2, 3$.

Ans As -2, and 3 are the roots of the required equation, thus

Sum of roots = S = –2 + 3 = 1

Product of roots = P = (–2)(3) = –6

The general equation to form quadratic equation is:

$$x^2 - Sx + P = 0$$

By putting the values of 'S' and 'P' in the above equation, we get

$$x^2 - (1)x + (-6) = 0$$

$$x^2 - x - 6 = 0$$

(v) Discuss the nature of roots of the equation:

$$x^2 + 3x + 5 = 0$$

Ans Given, $x^2 + 3x + 5 = 0$

Here, $a = 1,\ b = 3,\ c = 5$

Dicriminant $= b^2 - 4ac$

$$= (3)^2 - 4(1)(5)$$

$$= 9 - 20$$

$$= -11 < 0$$

So, the roots are imaginary (Complex Conjugates).

(vi) Find ω^2, if $\omega = \dfrac{-1+\sqrt{-3}}{2}$.

Ans If $\quad \omega = \dfrac{-1+\sqrt{-3}}{2}$

Then, by the properties of cube roots of unity

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

(vii) Find a, if the ratios a + 3 : 7 + a and 4 : 5 are equal.

Ans Since the ratios a + 3 : 7 + a and 4 : 5 are equal.

$\therefore$ in fraction form

$$\frac{a+3}{7+a} = \frac{4}{5}$$

$$5(a+3) = 4(7+a)$$

$$5a + 15 = 28 + 4a$$

$$5a - 4a = 28 - 15$$

$$a = 13$$

Thus, the given ratios will be equal if a = 13.

(viii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

(ix) Find a fourth proportional to: 5, 8, 15.

Ans Let x be the fourth proportional, then

$$5 : 8 :: 15 : x$$

$\therefore$ Product of extremes = Product of means

$$5(x) = (8)(15)$$

$$x = \frac{(8)(15)}{5}$$

$$x = 8 \times 3$$

$$x = 24$$

3. Write short answers to any SIX (6) questions: (12)

(i) Define identity.

Ans An identity is an equation which is satisfied by all the values of the variables involved.

(ii) Define complement of a set.

Ans Complement of a set A *w.r.t* universal set U is denoted by $A^C = A' = U - A$ contains all those elements of U which do not belong to A.

(iii) Find (A∩B) and (A∪B), when A = {2, 3, 5, 7} and B = {3, 5, 8}.

Ans As, $A \cap B = \{2, 3, 5, 7\} \cap \{3, 5, 8\}$

$= \{3, 5\}$

And, $A \cup B = \{2, 3, 5, 7\} \cup \{3, 5, 8\}$

$= \{2, 3, 5, 7, 8\}$

(iv) Find (A – B) and (B – A), when A = N and B = W.

Ans Given, $A = N$ and $B = W$

So, $A = \{1, 2, 3, \ldots\}$ and $B = \{0, 1, 2, 3, \ldots\}$

$\therefore\ A - B = \{1, 2, 3, \ldots\} - \{0, 1, 2, 3, \ldots\}$

$= \phi$

And B – A = {0, 1, 2, 3, ...} – {1, 2, 3, ... }
= {0}

(v) Write all subsets of A = {a, b}.

Ans All subsets of A = {a, b} are:

ϕ, {a}, {b}, {a, b}

(vi) The sugar contents for a random sample of 6 packs of juice of a certain brand are as given, find the median: 2.3, 2.7, 2.5, 2.9, 3.1 and 1.9.

Ans Arrange the values by increasing order of magnitude.
1.9, 2.3, 2.5, 2.7, 2.9, 3.1
Since number of observations are even, *i.e.*, n = 6.

$$\tilde{x} = \frac{1}{2}[\text{size of } (3^{rd} + 4^{th}) \text{ observations}]$$

$$= \frac{1}{2}[2.5 + 2.7]$$

$$= \frac{5.2}{2}$$

$$\tilde{x} = 2.6$$

(vii) Define variance.

Ans Variance is defined as the mean of the squared deviations of x_i (i = 1, 2, ..., n) observations from their arithmetic mean. In symbols, variance of X is:

$$Var(X) = S^2 = \frac{\Sigma(X - \bar{X})^2}{n}$$

(viii) Find the harmonic mean of a data: 12, 5, 8, 4

Ans

x	$\frac{1}{x}$
12	0.0833
5	0.2
8	0.125
4	0.25
	$\Sigma(\frac{1}{x}) = 0.6583$

$$H.M = \frac{n}{\Sigma(\frac{1}{x})}$$

$$= \frac{4}{0.6583} = 6.076$$

(ix) **Define mode.**

Ans Mode is defined as the most frequent occurring observation in the data. It is the observation that occurs maximum number of times in the given data. The following formula is used to determine mode:

For ungrouped data:

Mode = the most frequent observation

For grouped data:

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

4. Write short answers to any SIX (6) questions: (12)

(i) **Convert $\frac{3\pi}{4}$ to degrees.**

Ans

$$\frac{3\pi}{4} = \frac{3\pi}{4} \times 1 \text{ radian}$$

$$= \frac{3\pi}{4} \times \frac{180^\circ}{\pi}$$

$$= 135^\circ$$

(ii) **Find 'l', when $\theta = 180^\circ$ and $r = 4.9$ cm.**

Ans

$$\theta = 180^\circ$$

$$\theta = 180^\circ \times 1^\circ$$

$$\theta = 180^\circ \times \frac{\pi}{180^\circ}$$

$$\theta = \pi \text{ radians}$$

By putting,

$$l = r\,\theta$$

$$l = (4.9)\,\pi$$

$$l = \left(\frac{49}{10}\right)\left(\frac{22}{7}\right)$$

$$= \frac{154}{10} = \frac{77}{5}$$

$$l = 15.4 \text{ cm}$$

(iii) Define projection.

Ans The projection of a given point on a line is the foot of $\perp$ drawn from the point on that line. However, the projection of given point P on a line AB is the point P itself.

(iv) Define collinear points.

Ans Three or more points lying on the same line are called collinear points otherwise these are non-collinear points.

(v) Define secant.

Ans A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vi) Define sector of a circle.

Ans The sector of a circle is an area bounded by any two radii and the arc intercepted between them.

(vii) Define chord of a circle.

Ans The joining of any two points on the circumference of the circle is called its chord.

(viii) Define escribed circle.

Ans If a circle touches one side of a triangle externally and the other two produced sides internally, is called escribed circle.

(ix) Define vertices.

Ans The corners of a polygon are called its vertices.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: $2x^4 = 9x^2 - 4$. **(4)**

Ans Given: $2x^4 = 9x^2 - 4$

$$2x^4 - 9x^2 + 4 = 0$$

i.e., $$2(x^2)^2 - 9(x^2) + 4 = 0 \qquad \text{(i)}$$

Let, $y = x^2$, then (i) becomes

$$2y^2 - 9y + 4 = 0$$

By factorization:

$$2y^2 - 8y - y + 4 = 0$$

$$2y(y-4)-1(y-4)=0$$
$$(2y-1)(y-4)=0$$
$$2y-1=0 \quad ; \quad y-4=0$$
$$2y=1 \quad ; \quad y=4$$
$$y=\frac{1}{2}$$

Put $y=\frac{1}{2}$ in $y=x^2$, we get

$$y=x^2=\frac{1}{2}$$
$$\sqrt{x^2}=\pm\sqrt{\frac{1}{2}}$$
$$x=\pm\frac{1}{\sqrt{2}}$$

And put $y=4$ in $y=x^2$, we get

$$y=x^2=4$$
$$\sqrt{x^2}=\pm\sqrt{4}$$
$$x=\pm 2$$

Thus, solution set $=\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

(b) Show that the equation $x^2+(mx+c)^2=a^2$ has equal roots if $c^2=a^2(1+m^2)$. (4)

Ans Given, $x^2+(mx+c)^2=a^2$

$$x^2+(m^2x^2+c^2+2mcx)=a^2$$
$$x^2+m^2x^2+c^2+2mcx-a^2=0$$
$$(x^2+m^2x^2)+2mcx+(c^2-a^2)=0$$
$$(1+m^2)x^2+(2mc)x+(c^2-a^2)=0$$

As the above equation is in quadratic form, so

$$a=1+m^2, \quad b=2mc, \quad c=c^2-a^2$$

As the above equation has equal roots, so

$$\text{Discriminant}=0$$
$$b^2-4ac=0$$

By putting the values, we get

$$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$
$$4m^2c^2 - 4[1(c^2) + m^2(c^2) + 1(-a^2) + m^2(-a^2)] = 0$$
$$4m^2c^2 - 4[c^2 + m^2c^2 - a^2 - m^2a^2] = 0$$
$$4m^2c^2 - 4c^2 - 4m^2c^2 + 4a^2 + 4m^2a^2 = 0$$
$$-4c^2 + 4a^2 + 4m^2a^2 = 0$$
$$-4(c^2 - a^2 - a^2m^2) = 0$$
$$c^2 - a^2 - a^2m^2 = \frac{0}{-4}$$
$$c^2 - a^2 - a^2m^2 = 0$$
$$c^2 = a^2 + a^2m^2$$
$$\text{(Proved)} \quad c^2 = a^2(1 + m^2)$$

Q.6.(a) Using theorem of componendo-dividendo, solve: (4)

$$\frac{(x + 5)^3 - (x - 3)^3}{(x + 5)^3 + (x - 3)^3} = \frac{13}{14}$$

Ans By applying componendo-dividendo theorem, we get

$$\frac{\{(x + 5)^3 - (x - 3)^3\} + \{(x + 5)^3 + (x - 3)^3\}}{\{(x + 5)^3 - (x - 3)^3\} - \{(x + 5)^3 + (x - 3)^3\}} = \frac{13 + 14}{13 - 14}$$

$$\frac{(x + 5)^3 - (x - 3)^3 + (x + 5)^3 + (x - 3)^3}{(x + 5)^3 - (x - 3)^3 - (x + 5)^3 - (x - 3)^3} = \frac{27}{-1}$$

$$\frac{2(x + 5)^3}{-2(x - 3)^3} = -27$$

$$-\frac{(x + 5)^3}{(x - 3)^3} = -27$$

$$\frac{(x + 5)^3}{(x - 3)^3} = 27$$

$$\frac{(x + 5)^3}{(x - 3)^3} = (3)^3$$

By taking $\left(\frac{1}{3}\right)$ power on both sides, we have

$$\left[\frac{(x + 5)^3}{(x - 3)^3}\right]^{1/3} = [(3)^3]^{1/3}$$

$$\frac{(x+5)^{3 \times 1/3}}{(x-3)^{3 \times 1/3}} = 3^{3 \times 1/3}$$

$$\frac{x+5}{x-3} = 3$$

$$x + 5 = 3(x - 3)$$

$$x + 5 = 3x - 9$$

$$5 + 9 = 3x - x$$

$$14 = 2x$$

$$\frac{14}{2} = x$$

$\Rightarrow \quad x = 7$

Thus, the solution set = {7}.

(b) Resolve into partial fraction: **(4)**

$$\frac{1}{(x^2-1)(x+1)}$$

Ans For Answer see Paper 2018 (Group-II), Q.6.(b).

Q.7.(a) If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9}, B = {2, 3, 5, 7}, then verify that $(A \cap B)' = A' \cup B'$. **(4)**

Ans L.H.S = $(A \cap B)'$

So,

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$

$A \cap B = \{3, 5, 7\}$

$(A \cap B)' = U - (A \cap B) = \{1, 2, 3, \ldots 10\} - \{3, 5, 7\}$

$(A \cap B)' = \{1, 2, 4, 6, 8, 9, 10\}$

R.H.S = $A' \cup B'$

$A' = U - A = \{1, 2, 3, \ldots, 10\} - \{1, 3, 5, 7, 9\}$

$A' = \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$

$B' = \{1, 4, 6, 8, 9, 10\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$

$A' \cup B' = \{1, 2, 4, 6, 8, 9, 10\}$

So it is proved that

L.H.S = R.H.S

$(A \cap B)' = A' \cup B'$

(b) The marks of six students in Mathematics are as followed. Determine variance: **(4)**

Students	**1**	**2**	**3**	**4**	**5**	**6**
Marks	**60**	**70**	**30**	**90**	**80**	**42**

Ans For Answer see Paper 2017 (Group-II), Q.7.(b).

Q.8.(a) Prove that: $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \dfrac{\sin\theta}{1-\cos\theta}$. **(4)**

Ans L.H.S $= \sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} \quad \text{By Rationalization}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}}$$

$$= \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{1-\cos^2\theta}}$$

$$= \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} \qquad \because \ 1-\cos^2\theta = \sin^2\theta$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

Again,

$$= \frac{1+\cos\theta}{\sin\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1)^2-(\cos\theta)^2}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta\,(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta\,(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

$= R.H.S$ (Proved)

(b) Inscribe a circle in an equilateral triangle ABC with each side of length 5 cm. **(4)**

Ans For Answer see Paper 2017 (Group-II), Q.8.(b).

Q.9. Prove that perpendicular from the centre of a circle on a chord bisects it. **(4)**

Ans **Given:**

$\overline{AB}$ is the chord of a circle with centre at O so that $\overline{OM} \perp$ chord $\overline{AB}$.

Prove:

M is the mid-point of chord $\overline{AB}$ *i.e.,* $m\overline{AM} = m\overline{BM}$.

Construction:

Join A and B with centre O.

Proof:

Statements	Reasons
In $\angle$rt Δ^s OAM $\leftrightarrow$ OBM	
$m\angle OMA = m\angle OMB = 90^\circ$	Given
Hyp. $\overline{OA}$ = Hyp. $\overline{OB}$.	Radii of the same circle
$m\overline{OM} = m\overline{OM}$	Common
$\therefore$ $\Delta OAM \cong \Delta OBM$	In $\angle$rt Δ^s H.S $\cong$ H.S
Hence, $m\overline{AM} = m\overline{BM}$	
$\Rightarrow$ $\overline{OM}$ bisects the chord $\overline{AB}$.	

Corollary 1:

$\perp$ bisector of the chord of a circle passes through the centre of a circle.

Corollary 2:

The diameter of a circle passes through the mid-points of two parallel chords of a circle.

OR

Prove that the measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Ans For Answer see Paper 2017 (Group-I), Q.9.(OR).