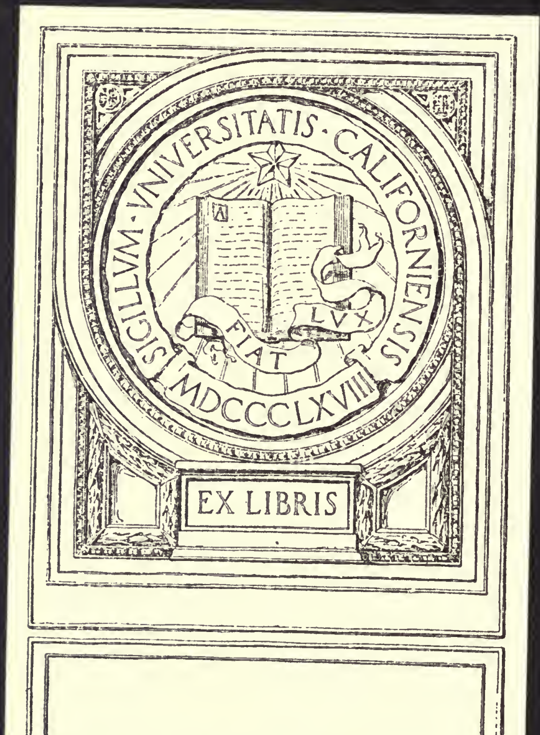
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Reinforced Concrete Building Design

by

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Instructor of Bridge Engineering, Cornell University

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INTRODUCTION

The following pages are intended to furnish the computing side of an elementary course in reinforced concrete construction and to supplement a text book used in the class room. The references made to Diagrams and Tables are to those of *Reinforced Concrete Construction*, Volume I, by George A. Hool.

It has been the intention to give a complete design for the roofs, floors, columns, and footings of a reinforced concrete building of the beam-and-girder type. Other details such as walls, windows, stairways, elevator shafts, etc., do not as a rule affect the main design, involve more practical considerations, and will not be taken up here. For an example of these details the student is referred to Volume II of the text book mentioned above.

The design of the floors and columns is given in a very precise form, and although so much exactness is not required in ordinary building design, it has been the intention to familiarize the student with methods applicable to large and important members, and also to show methods of checking each portion of the design.

There are many methods of form construction, but the one briefly given in the following design seems to be more general than many others. Here again more exactness will be found than is ordinarily required, but in buildings having many similar units such exactness would result in final economy.

The design of a combined footing is given in order to furnish an outline for a problem of this type in the computing room.

CORNELL UNIVERSITY,
October 31, 1915.

SPECIFICATIONS

1. The concrete in this structure shall consist of a wet mixture of one part Portland cement to six parts of aggregate, fine and coarse, and shall be capable of developing an average compressive stress of 2000 pounds per square inch at twenty-eight days when tested in cylinders eight inches in diameter and sixteen inches long under laboratory conditions of manufacture.

2. All reinforcement shall consist of medium open hearth steel having an elastic limit of not less than 50000 pounds per square inch, and shall be of plain round section.

3. All flexure formulae shall be based on the "straight line theory."

4. The span lengths used in computations shall be taken as the distance center to center of supports.

5. Slabs, beams, and girders shall be considered as fully continuous, and bending moments shall be computed accordingly both for uniform and concentrated loads.

6. The negative moment which exists at the supports shall be taken equal to the maximum positive moment sustained by the slab, beam, or girder.

Reinforcement shall be provided to resist this negative moment and all such reinforcement shall be continued beyond the support to the quarter point of the adjacent span, but in no case shall this distance be less than that required to develop the full strength of the rods at the allowable bond stress.

7. The lateral spacing of parallel rods shall not be less than two and one-half diameters center to center, nor shall the distance from the side of the beam or girder to the center of the nearest rod be less than two diameters. Layers of rods, in beams and girders, shall have at least one-half inch vertically in the clear. In girders and columns the metal shall be protected by a minimum of two inches of concrete; (i. e. from the surface of the rod to the surface of the concrete) in beams, by one and one-half inches; in floor slabs, by one-half inch of concrete. Main reinforcement in slabs, beams, and girders shall not be less than three-eighths of an inch nor more than one inch in diameter. Stirrups shall not be less than one quarter inch in diameter.

8. In columns the concrete to a depth of one and one-half inches shall be considered as protective covering and not included in the effective section.

9. The minimum allowable thickness for slabs is four inches, and the main reinforcement when consisting of rods shall not be placed farther apart than two-and-one-half times the slab thickness.

Rods placed perpendicular to the main reinforcement for the prevention of cracks, due to shrinkage, etc., shall be provided to an amount not less than one-fourth of one percent of the slab section.

10. Beams whose axes are perpendicular to the main reinforcement of the slab may be designed as Tee beams, provided the slab is poured with the beams and properly bonded to the same. Under these conditions the effective width of flange shall be taken so as not to exceed (a) one-fourth of the span length of the beam, (b) eight times the thickness of the slab plus the width of the stem.

11. When the maximum shearing stresses exceed the value allowed for the concrete alone, web reinforcement must be provided to aid in carrying the diagonal tension stresses.

The horizontal reinforcement is to be bent up where possible to assist in taking the diagonal tension stresses, but this is not to be considered in determining the required amount of web reinforcement.

In the calculations of web reinforcement the concrete may be counted upon as carrying one-third of the shear, and the remainder is to be provided for by means of metal reinforcement.

The maximum longitudinal spacing of vertical stirrups shall not exceed three-fourths of the working depth of the beam. The effective length of stirrups in bond shall be taken as six-tenths of the working depth of the beam.

12. The ratio of length to diameter of concrete columns shall not exceed fifteen.

Columns shall not have less than one per cent. nor more than four per cent. of longitudinal reinforcement, which shall be held together by bands or hoops, not less than one-fourth inch in diameter, spaced center to center not more than one-half of the diameter of the column.

Not less than four rods shall be used for the reinforcement of any column, and rods shall not be less than one-half inch nor more than one inch in diameter.

Bending stresses due to eccentric loads must be provided for by increasing the section until the maximum stress does not exceed the allowable value.

13. The footings shall be so designed that they can be poured at one operation. The bottom of the footing shall not be nearer to the lowest layer of steel than four inches.

14. The allowable working stresses shall not exceed the following:

Axial compression on columns reinforced with longitudinal steel and spirals or bands .... 500 pounds per square inch
Concrete in compression in slabs, beams, or girders .... 650 pounds per square inch
Concrete in compression over the support of continuous slabs, beams and girders............ 750 pounds per square inch
Shearing strength of plain concrete as a measure of diagonal tension .... 40 pounds per square inch
Shearing strength of concrete with proper metal reinforcement .... 120 pounds per square inch
Bond stress between concrete and plain reinforcing rods .... 80 pounds per square inch
Tensile stress in reinforcing steel....16000 pounds per square inch

Compressive stress in reinforcing steel not more than fifteen times the working compressive stress in the concrete.

Ratio of the modulus of elasticity of steel to that of concrete....15

The building whose partial design is given in the following pages has three stories above ground and a basement, and has the following dimensions:

Width, center to center of wall columns	54′0″
Length, center to center of wall columns	100′0″
Span of slabs	9′ 0″
Span of beams	20′ 0″
Span of girders	18′ 0″
Distance floor to floor	12′ 0″

It is designed for a live load on the floors of 100 pounds per square foot, a live load on the roof of 35 pounds per square foot, and the allowable soil pressure is taken as 4000 pounds per square foot.

DESIGN OF THE FLOOR SLAB

Assumed thickness from Table 6 . . . . 4 inches
Total load per square foot . . . . 150 pounds
Bending moment for unit slab $\frac{1}{12} \times 150 \times 9^2 \times 12 = 12150$ in-lbs.

From Table 2 p = ...0077
k = ...368
j = ...874
K = ... 107.4

$$d = \sqrt{\frac{12150}{107.4 \times 12}} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.07 \text{ inches}$$

Taking the value of *d* to the nearest quarter of an inch greater than the theoretical value determined, and adding either three quarters of one inch or one inch for insulation, so that the total thickness of the slab is determined to the nearest one-half inch, the actual value of *d* is 3¼ inches and the total thickness of the slab 4 inches as assumed.

$$a_s = \frac{12150}{16000 \times .874 \times 3.25} = \ldots\ldots\ldots\ldots\ldots .267 \text{ square inches}$$

$$\varepsilon_o = \frac{150 \times 4.5}{80 \times .874 \times 3.25} = \ldots\ldots\ldots\ldots\ldots\ldots 2.97 \text{ inches}$$

Using $\frac{3}{8}$ inch round rods $s = \frac{.1104}{.267} \times 12 = \ldots\ldots$ 4.96 inches

$$\text{or } s = \frac{1.178}{2.97} \times 12 = \ldots\ldots 4.76 \text{ inches}$$

Rods will be spaced . . . . $4\frac{1}{2}$ inches center to center.

To provide for the negative bending moment near the support each alternate rod is bent upward near the quarter point of the span and continued over the support to the quarter point of the adjacent span. This distance beyond the support must not be less than that required to develop the full strength of the rod in bond which is $a_s f_s \div$ perimeter of rod x u: for a $\frac{3}{8}$ inch rod the distance is $18\frac{3}{4}$ inches which is less than one-quarter of the span (27 inches).

The reinforcement required in the other direction is .0025 x 4 x 12 = .12 square inches per foot section of slab, requiring that $\frac{3}{8}$ inch round rods be placed $\frac{.1104}{.120} \times 12 = 11.04$ or 11 inches center to center.

REVIEW OF THE FLOOR SLAB

$$p = \frac{.1104}{4.5 \times 3.25} = \ldots\ldots\ldots\ldots\ldots \quad .0075$$

From Table 3 $k = \ldots\ldots\ldots\ldots\ldots \quad .375$

$j = \ldots\ldots\ldots\ldots\ldots \quad .875$

a_s for a unit slab $= \frac{12}{4.5} \times .1104 \ldots\ldots$ .294 square inches

$$f_s = \frac{12150}{.294 \times .875 \times 3.25} = \ldots\ldots\ldots\ldots\ldots \quad 14500$$

$$f_c = \frac{2 \times 14500 \times .0075}{.375} = \ldots\ldots\ldots\ldots\ldots \quad 580$$

$$K = \frac{12150}{12 \times 3.25^2} = \ldots\ldots\ldots\ldots\ldots \quad 95.9$$

From Diagram 1, with the value of $K = 95.9$ and $p = .0075$, $f_s = 14500$ and $f_c = 580$ which check the above computations.

$$v = \frac{150 \times 4.5}{12 \times .875 \times 3.25} = \ldots\ldots\ldots\ldots\ldots \quad 19.8$$

$$u = \frac{150 \times 4.5}{\frac{12 \times 1.178}{4.5} \times .875 \times 3.25} = \ldots\ldots\ldots\ldots\ldots \quad 76.5$$

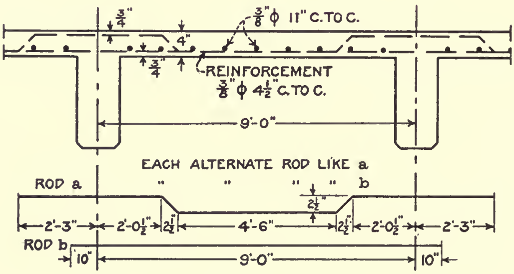

FIGURE 1

DESIGN AND REVIEW OF THE FLOOR BEAM

The weight of slab carried by one floor beam is 50 x 20 x 9 = ... 9000 pounds

The live load carried by one floor beam is 100 x 20 x 9 = 18000 pounds

The weight of the beam below the slab may be assumed as 10% of the above weight = ... 2700 pounds

29700 pounds

The bending moment is $\frac{1}{12} \times 29700 \times 20 \times 12 = 594000$ inch pounds

In order that the allowable unit ~~shearing stress~~ shall not be exceeded, the beam must have a cross-section $b'd$ equal to $\frac{29700 \div 2}{105}$ $= 141.4$ square inches. With $b' = 7\frac{3}{4}$ inches, $d = 18.25$ inches. For economy with $b' = 7\frac{3}{4}''$ and the ratio of cost between the steel and concrete taken as 60, the depth d should be $\sqrt{\frac{60 \times 594000}{16000 \times 7\frac{3}{4}}} + \frac{4}{2}$ $= 18.95$ inches. Therefore d must be equal to or greater than 18.25 inches and as nearly equal to 18.95 inches as the forms will permit. (See actual sizes of lumber under *Design of Forms*). The

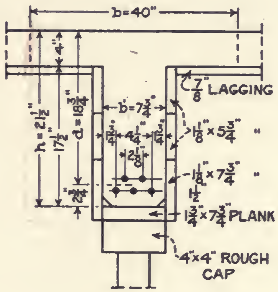

FIGURE 2

distance to be covered by the side lagging (see Figure 2) must then be at least $18.25 + 2\frac{3}{4} + 1\frac{3}{4} - 4 = 18.75$ inches. Two pieces $1\frac{1}{8}'' \times 5\frac{3}{4}''$ and one piece $1\frac{1}{8}'' \times 7\frac{3}{4}''$ furnish $19\frac{1}{4}$ inches; d is taken as $18\frac{3}{4}$ inches, and the total depth of the beam $18\frac{3}{4}'' + 2\frac{3}{4}'' = 21\frac{1}{2}$ inches. The distance of $2\frac{3}{4}$ inches allows for two rows of rods, whose centers are $1\frac{1}{2}$ inches apart vertically, and assuming that the rods are not larger than one inch in diameter, this allows a clear distance below the lower rods of $1\frac{1}{2}$ inches according to the specifications.

The actual weight of the beam below the slab is

$$\frac{17\frac{1}{2} \times 7\frac{3}{4} \times 150 \times 20}{144} = \ldots \quad 2830 \text{ pounds}$$

The revised total load on the beam.............	29830 pounds
The revised bending moment	596600 inch pounds
The revised maximum shear	14915 pounds

Taking the value of jd equal to $d - \frac{1}{2}t$,

$$a_s = \frac{596600}{16\frac{3}{4} \times 16000} = 2.23 \text{ sq. in. and } \varepsilon_o = \frac{14915}{16\frac{3}{4} \times 80} = 11.13 \text{ sq. in.}$$

Six $\frac{3}{4}$ inch round rods furnish an area of 2.65 square inches and a total perimeter of 14.14 inches, but over the support the stresses in the steel and in the concrete cannot be brought within the allowable value unless all of the lower rods are carried through to the quarter point of the adjacent span. Five $\frac{7}{8}$ inch round rods, furnishing very little more steel do not require these long lower rods and are adopted. The latter furnish an area of 3.01 square inches and a total perimeter of 13.75 inches.

The width of the flange (b) is taken as large as the specifications will permit, i. e. either $\frac{1}{4} \times 20'-0'' = 60$ inches, or $8 \times 4 + 7\frac{3}{4} = 39.75$ inches; the latter value (40 inches) governing in this case.

$p = \dfrac{3.01}{18.75 \times 40} = .0040$ and $\dfrac{t}{d} = \dfrac{4}{18\frac{3}{4}} = .21.$ On Diagram 9 the point located by the above values of p and $\dfrac{t}{d}$ falls above the dotted line showing that the neutral axis is in the web.

From Table 9, with $\dfrac{t}{d} = .21$ and $p = .0040$

$k = .304$ $\qquad\qquad$ $j = .914$

From Diagram 9, $k = .30$ and $j = .91$, thus checking the above values from Table 9.

$$f_s = \frac{596600}{3.01 \times .914 \times 18.75} = \ldots\ldots\ldots\ldots \quad 11600$$

$$f_c = \frac{.314 \times 11600}{15\,(1 - .304)} = \ldots\ldots\ldots\ldots \quad 338$$

Three rods from each beam are bent up and carried over the support to the quarter point of the span of the adjacent beam. One of these rods from each beam reaches its upper position directly over the center of the support so that these two rods together only have the effective strength of one rod. Thus with three rods from each beam bent up the effective steel area over the support is equal to five $\frac{7}{8}$ inch round rods or 3.01 square inches. The two rods from each beam which are not bent up are carried beyond the ends of the rods from the adjacent beam far enough to develop the full strength of the rods in bond, (43.75 inches for a $\frac{7}{8}$ inch rod) making the effective steel area at the bottom of the beam over the support equal to two $\frac{7}{8}$ inch round rods or 1.20 square inches. In the case of the six $\frac{3}{4}$ inch round rods mentioned above it would have been necessary to carry three rods from each beam through along the bottom to the quarter point of the adjacent span in order to obtain a large enough steel area to keep the unit stresses within the allowable limits.

(For beams having an even number of rods, one-half of the total number of rods from each beam can be bent up and carried through to the quarter point of the adjacent beam, while the remaining half is carried past the rods from the adjacent beam far enough to develop the full strength of the rods in bond. This gives an effective area at the top of the beam equal to the total number of

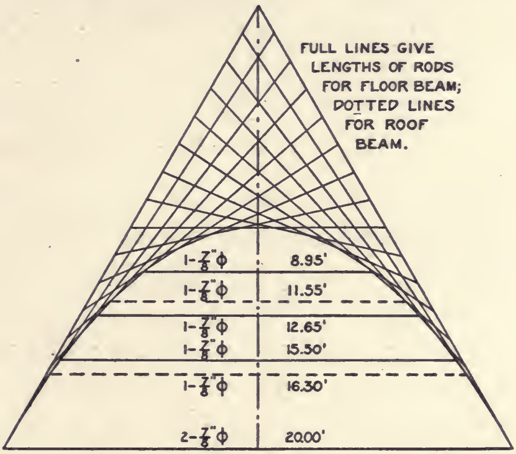

FIGURE 3

rods in one beam, and an effective area at the bottom equal to one-half of this amount.)

The points at which the three rods may be bent up are determined as follows:

One rod at $\frac{20}{2}\left(1-\sqrt{\frac{1}{5}}\right) = 5.53$ feet from the support.

One rod at $\frac{20}{2}\left(1-\sqrt{\frac{2}{5}}\right) = 3.68$ feet from the support.

One rod at $\frac{20}{2}\left(1-\sqrt{\frac{3}{5}}\right) = 2.26$ feet from the support.

These points are also checked graphically by means of Figure 3.

The upper layer of rods over the support is placed as high as the slab rods will permit so that their centers may be considered to be $1\frac{1}{2}$ inches from the upper surface of the slab.

$b =$ .... $7\frac{3}{4}$ inches

$d = 21\frac{1}{2} - 1\frac{1}{2} - \frac{3}{4}$ .... $19\frac{1}{4}$ inches

$d' =$ .... $2\frac{3}{4}$ inches

$a_s =$ .... 3.01 sq. in.

$a'_s =$ .... 1.20 sq. in.

$p = \dfrac{3.01}{7\frac{3}{4} \times 19\frac{1}{4}} =$ .... .0204

$p' =$ .... .0082

$\dfrac{p'}{p} =$ 4

$\dfrac{d'}{d} =$ .... .143

(See Figure 4)

From Table 11, interpolating for $p = .0204$

	$p' = .25p$			$p' = .50p$			$p' = .4p$		
	L	K	j	L	K	j	L	K	j
$\frac{d'}{d} = .10$	.262	.0173	.848	.305	.0177	.864	Interpolating		
$\frac{d'}{d} = .15$	.255	.0169	.836	.288	.0172	.845			
$\frac{d'}{d} = .143$	.256	.0170	.838	.291	.0173	.848	.277	.0172	.844

$f_c = \dfrac{596600}{7\frac{3}{4} \times (19\frac{1}{4})^2 \times .277} =$ 749

$f_s = \dfrac{596600}{7\frac{3}{4} \times (19\frac{1}{4})^2 \times .0172} =$ 12100

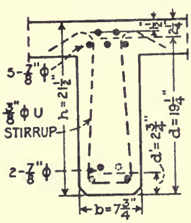

FIGURE 4

The value of *jd* at the midspan is $.916 \times 18\frac{3}{4} = 17.18$

The value of *jd* at the support is $.844 \times 19\frac{1}{4} = 16.25$

The value of *jd* would of course change at the point of inflection but the design will be on the side of safety if the smaller value of *jd* computed above is used in all of the computations involving shear and bond.

The maximum shear at the support is .... 14915 pounds
The maximum shear at midspan is $\frac{1}{8} \times 18000 =$ .. 2250 pounds

The unit shear at the support is $\frac{14915}{7\frac{3}{4} \times 16.25} =$ .. 118.5 pounds

The unit shear at midspan is $\frac{2250}{7\frac{3}{4} \times 16.25} =$ ..... 17.9 pounds

The distance from the support beyond which stirrups are not required (i. e. the point where the unit shear is equal to 40 pounds per square inch) may be determined by the equation $\frac{w_l x^2}{2l} + w_d x = 40bjd + \frac{w_d l}{2}$, where x = the distance from the section to the opposite support, w_d = the dead load per linear foot and w_l = the live load per linear foot.

$w_l = 100 \times 9 = 900$ pounds, $w_d = 50 \times 9 + 142 = 592$ pounds

$\frac{900x^2}{2 \times 20} + 592x = 40 \times 7\frac{3}{4} \times 16.25 + \frac{592 \times 20}{2}$, $x^2 + 26.32\,x = 487.0$, and $x = 12.53$ feet, so that stirrups are required over a distance of $20 - 12.53 = 7.47$ feet from the support. Selecting a stirrup that requires a spacing of about 6 inches at the support, $a_s = \frac{2}{3} \times \frac{14915 \times 6}{16000 \times 16.25} = .230$ square inches; a $\frac{3}{8}$ inch round U stirrup furnishes .2208 square inches and is adopted.

The maximum shear at any section is $\frac{w_l x^2}{2l} + w_d\,(x - \frac{l}{2})$.

$\frac{w_l}{2l} = 22.5$ $\frac{w_d l}{2} = 5920$

Dis. from Support	x	x^2	$\frac{w_l x^2}{2l}$	$w_d x$	V	$\frac{3}{2} a_s f_s jd$	Spacing inches
0	20	400	9000	11840	14920	86100	5.8
1	19	361	8120	11250	13450	86100	6.4
2	18	324	7290	10660	12030	86100	7.2
3	17	289	6500	10060	10640	86100	8.1
4	16	256	5760	9470	9310	86100	9.2
5	15	225	5060	8880	8020	86100	10.7
6	14	196	4410	8290	6780	86100	12.7
7	13	169	3800	7700	5580	86100	15.4
7.47	12.53	157	3530	7420	5030	86100	17.1

The maximum spacing allowed by the specifications is $\frac{3}{4} \times 18\frac{3}{4} = 14.06$ inches so that in drawing a curve for the stirrup spacings similar to Figure 46 of the text, it becomes a horizontal line when its ordinate reaches a value of 14 inches. In drawing this curve it is necessary that the horizontal and vertical scales be the same in order that the spacings plotted as ordinates may be transferred to the horizontal axis by means of 45° lines. These lines cannot be drawn until the location of the first stirrup is determined, which will be placed two or three inches from the edge of the support, and the width of the column or girder which supports the floor beam is not yet known.

The bond stress developed on the stirrups is $\frac{.2208 \times 16000}{2 \times 1.178 \times .6 \times 18.75}$ = 133 pounds per square inch. This high value is safe, however, since the upper ends of the stirrups are bent into the form of hooks.

The bond stress on the upper horizontal rods at the support is $\frac{14915}{5 \times 2.749 \times 16.25}$ = 66.8 pounds per square inch and is within the allowable value.

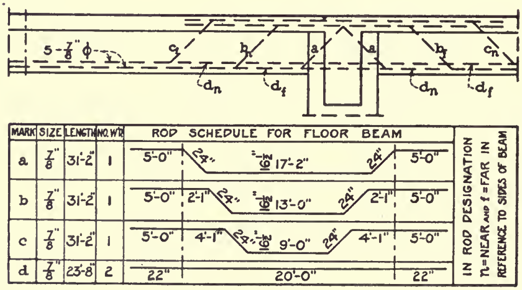

FIGURE 5

DESIGN AND REVIEW OF GIRDER.

The maximum moment in the girder has the same value when all of the load in one bay is considered as uniformly distributed on the girder, as it does when the load from one floor beam is considered as concentrated at the center of the girder. The total load supported by two floor beams is 59660 pounds, and the weight of the girder below the slab may be assumed as 11tLS, where L is the span of the beam and S the span of the slab. This assumed weight is $11 \times 4\frac{1}{2} \times 20 \times 9 = 8910$ pounds, and the total load is 68570 pounds.

$$M = \frac{1}{12} \times 68570 \times 18 \times 12 = \dots\dots\dots\dots \ 1234260 \text{ inch pounds}$$

$$bd^2 = \frac{1,234,260}{107.4} = \dots\dots\dots\dots\dots\dots \ 11500$$

With $b = 13\frac{1}{2}$ inches, d must be at least 29.2 inches.

The distance to be covered by the side lagging must be at least $29.2 + 2\frac{1}{2} + 1\frac{3}{4} - 4 = 29.45$ inches (see Figure 6). Four pieces $7\frac{3}{4} \times 1\frac{1}{8}$ furnish 31 inches; d is taken as 30.75 inches and the total depth of the beam (h) as 33.25 inches.

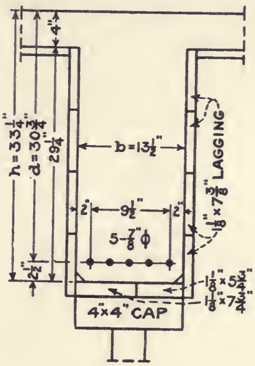

FIGURE 6

The actual weight of the beam below the slab is $\frac{29\frac{1}{4} \times 13\frac{1}{2}}{144} \times 150 \times 18 = 7410$ pounds, the revised total load 67070 pounds, and the revised bending moment 1207260 inch pounds.

$$a_s = \frac{1207260}{16000 \times .874 \times 30.75} = 2.81 \text{ square inches.}$$

Five $\frac{7}{8}$ inch round rods furnish an area of 3.01 square inches and can be placed in one row as assumed.

$$p = \frac{3.01}{13\frac{1}{2} \times 30.75} = \dots\dots\dots\dots\dots\dots\dots\dots \ .0072$$

Table 3, $k = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ .369

$j = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ .877

$$f_s = \frac{1,207,260}{3.01 \times .877 \times 30.75} = \dots\dots\dots\dots\dots \ 14900$$

$$f_c = \frac{2 \times .0072 \times 14900}{.369} = \dots\dots\dots\dots\dots \ 582$$

$$K = \frac{1,207,260}{13.5 \times 30.75^2} = \dots\dots\dots\dots\dots \ 94.6$$

From Diagram 1 with $K = 94.6$ and $p = .0072$, $f_s = 15000$ and $f = 580$, which check the above computations.

In order more nearly to approach the actual conditions of load distribution in the girder, it is assumed that the girder supports the slab and its live load for a distance on each side of the girder equal to its own width, and the load carried by one floor beam not included in the above is assumed as concentrated at the center of the girder. Accordingly,

(1) The uniform load $= \frac{3 \times 13\frac{1}{2}}{12} \times 150 \times 18 + 7410 = 16520$ pounds

(2) The concentrated load $= 29830 - 9110 \div 2 = 25275$ pounds

$M_u = \frac{1}{12} \times 16520 \times 18 \times 12 =$ . . . . 297360 inch pounds

$M_c = \frac{1}{6} \times 25275 \times 18 \times 12$ . . . . 909900 inch pounds

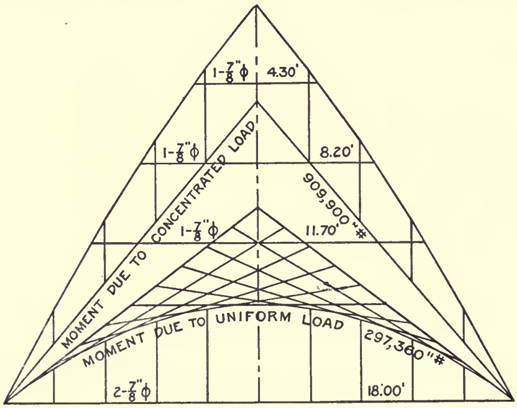

FIGURE 7

Using these two moments Figure 7 is constructed and the points at which the rods may be bent up are determined. Both Figures 3 and 7 neglect the continuity of the construction and therefore give very conservative results.

As both the bending moment and the cross-section of the beam have the same value at the support as at midspan, five rods will be required at the top of the beam over the support, while none is required at the bottom. However, it is better design to carry the lower rods through in a manner similar to that of the floor beam.

Maximum shear at the support $\frac{25275 + 16520}{2} = 20900$ pounds

Maximum shear at midspan $\frac{25275}{2} = \ldots\ldots\ldots\ldots$ 12640 pounds

Unit shear at the support $\frac{20900}{13.5 \times 30.75 \times .877} = \ldots$ 57.4 pounds

Unit shear at midspan $\frac{12640}{13.5 \times 30.75 \times .877} = \ldots\ldots$ 34.7 pounds

The true maximum shear at midspan would occur when a strip equal to three times the width of the girder times the span of the slab is unloaded, which would give a value of 13020 pounds, but the difference is so small that it may be neglected and the shear may be assumed to vary uniformly throughout the span.

The point beyond which stirrups are not required is $\frac{(40 - 34.7)\,9}{57.4 - 34.7}$ $= 2.10$ feet from midspan. Selecting a stirrup which requires a spacing of about 6 inches at the support $a_s = \frac{\frac{2}{3} \times 20900 \times 6}{16000 \times 30.75 \times .877}$ $= .194$ square inches; a $\frac{3}{8}$ inch round U stirrup furnishes .2208 square inches and is adopted.

The spacing required at the support is $\frac{\frac{3}{2} \times .2208 \times 16000}{57.4 \times 13\frac{1}{2}} = 6.8$ inches and that at a point 2.10 feet from midspan 9.8 inches. A curve similar to that used for the floor beam may be constructed by connecting the points located by the two spacings computed above by a straight line, and the actual spacing of the stirrups determined when the width of the column is known.

The actual bond stress on the horizontal rods is

$$\frac{20900}{5 \times 2.749 \times .877 \times 30.75} = 56.4 \text{ pounds per square inch.}$$

A rod schedule similar to Figure 5 can now be constructed for the girder.

DESIGN OF THE ROOF.

The design of the roof is similar to that of the floor, but as the loads are smaller, the construction will be somewhat lighter and only the design will be considered, the construction not being heavy enough to call for a review.

Roof Slab. Assuming the minimum allowable thickness of slab which is four inches, the total load carried is $50 + 35 = 85$ pounds per square foot. The bending moment for a unit slab is 6885 inch pounds and the required depth to the steel 2.31 inches. Using a depth as large as the thickness of the slab will permit (i. e. 3.25 inches), the steel area required per unit slab is .152 square inches, and $\frac{3}{8}$ inch round rods spaced $8\frac{1}{2}$ inches center to center furnish this area. The reinforcement required in the other direction is .12 square inches per foot section which is furnished by $\frac{3}{8}$ inch round rods spaced 11 inches center to center. The main reinforcing rods are bent upward at the same points as those used in the floor slab.

Roof Beam. The total weight of slab and live load sustained by the beam is $85 \times 20 \times 9 = 15300$, and assuming the weight of the beam as 10% of this load the total assumed load is 16830 pounds, the end shear 8415 pounds, and the bending moment 336600 inch pounds. The required value of $b'd$ for shear is 80.2 square inches and with $b' = 5\frac{3}{4}$ inches, d must be at least 13.9 inches. The economical value of d is 16.8 inches, and the distance to be covered by the side lagging should be somewhat more than $13.9 + 2\frac{3}{4} + 1\frac{3}{4} - 4 = 14.4$ inches. Two pieces $1\frac{1}{8}'' \times 7\frac{3}{4}''$ furnish 15.5 inches making the value of d 15.0 inches and the actual weight of the beam below the slab $\frac{13\frac{3}{4} \times 5\frac{3}{4}}{144} \times 150 \times 20 = 1650$ pounds. The revised values of the total load, end shear, and bending moment are 16950 pounds, 8475 pounds, and 339,000 inch pounds respectively.

The area of steel required is 1.63 square inches and the total perimeter 8.15 inches, which can be furnished by three $\frac{7}{8}$ inch round rods with an area of 1.80 square inches and a total perimeter of 8.25 inches.

The points at which two of the rods may be bent up can be determined by the use of Figure 3, the dotted lines representing the length of rods for the roof beam.

It will be assumed that an effective area of steel at the top of the beam over the support of three $\frac{7}{8}$ inch round rods and one $\frac{7}{8}$ inch round rod at the bottom of the beam will be sufficient to keep the unit stresses within the allowable values. Figures similar to 2, 4, and 5 can now be constructed for the roof beam.

The maximum shear at the center of the beam is $\frac{1}{8} \times 6300 = 788$ pounds, and taking $jd = d - \frac{1}{2}t = 13$ inches, the unit shear at the support is 113.5 pounds and at midspan 10.5 pounds. Since the live load is small it may be assumed that the shear varies uniformly. Then the point beyond which stirrups are not required is $\frac{10\,(40 - 10.5)}{113.5 - 10.5} = 2.86$ feet from midspan or 7.14 feet from the support. By using the same method of finding the location of this point as was used in the design of the floor beam, this distance is found to be 7.03 feet from the support so that the error in the above computation is small and on the side of safety.

Selecting a stirrup that requires a spacing of about six inches at the support $a_s = \frac{2}{3} \times \frac{8475 \times 6}{16000 \times 13} = .163$ square inches; a $\frac{5}{16}$ inch round U stirrup furnishes .1534 square inches and is adopted. The spacing required at the support is 5.6 inches and at the point 7.14 feet from the support 16.0 inches. According to the specifications 11.25 inches is the maximum allowable distance between stirrups. With these spacings as ordinates a curve of stirrup spacings can be constructed similar to that of the floor girder.

Roof Girder. The total load sustained by two roof beams is 33900 pounds and assuming that the weight of the girder below the slab is $3.2LS^2$ pounds = 5180 pounds, the total assumed load is 39080 pounds, and the bending moment 703440 inch pounds. Then $bd^2 = 6550$, and with $b = 11\frac{3}{4}$ inches, d must be at least 23.6 inches, and the distance to be covered by the side lagging must be at least $23.6 + 2\frac{1}{2} + 1\frac{3}{4} - 4 = 24.1$ inches. Three pieces $1\frac{1}{8}'' \times 5\frac{3}{4}''$ and one piece $1\frac{1}{8}'' \times 7\frac{3}{4}''$ furnish 25 inches, making $d = 24.5$ inches and the actual weight of the girder below the slab $\frac{23 \times 11\frac{3}{4}}{144} \times 150 \times 18 =$ 5070 pounds, and the revised total load 38970 pounds.

The revised bending moment is 701,460 inch pounds, and the steel area required is 2.05 square inches, which is furnished by four $\frac{7}{8}$ inch round rods, which can all be placed in one row, as assumed.

Assuming the same distribution of loads as in the floor girder, the uniform load is $\frac{3 \times 11\frac{3}{4}}{12} \times 85 \times 18 + 5070 = 9570$ pounds, and the concentrated load $16950 - \frac{4500}{2} = 14700$ pounds, the bending moment due to the uniform load 172,260 inch pounds, and that due to the concentrated load 529,200 inch pounds. With these moments a diagram similar to Figure 7 is constructed and it is found that two of the rods may be bent up at distances of 6.45 feet and 4.05 feet respectively from the support.

The maximum shear at the support is 12135 pounds and the unit shear 48.2 pounds, and the corresponding values at midspan are 7350 pounds and 29.2 pounds respectively. The point beyond which stirrups are not required is 3.36 feet from the support. A $\frac{5}{16}$ inch round U stirrup requires a spacing of 6.5 inches at the support and a spacing of 7.8 inches at a point 3.36 feet from the support. The difference between these spacings is so slight that no diagram need be constructed and the stirrups may be spaced 7 inches center to center over the distance they are required. Figures similar to 5 and 6 can now be constructed for the roof girder.

COLUMNS.

Two columns are to be designed: one, the typical interior column supporting the main floor, and being in turn supported by the basement footing; two, an exterior column or wall column supporting the main floor and having a girder framing into it whose axis is perpendicular to the wall. In low buildings the columns are often made the same size throughout, but in high buildings large quantities of material may be saved by decreasing the sizes of the columns toward the top of the building. The same general method of design applies.

The eccentric moment in the column due to a possible eccentric load on the floor is to be taken equal to one-fourth of the moment produced by the eccentric load in the girder framing into the column. This reduction is allowable on account of the stiffness and the continuity of the construction.

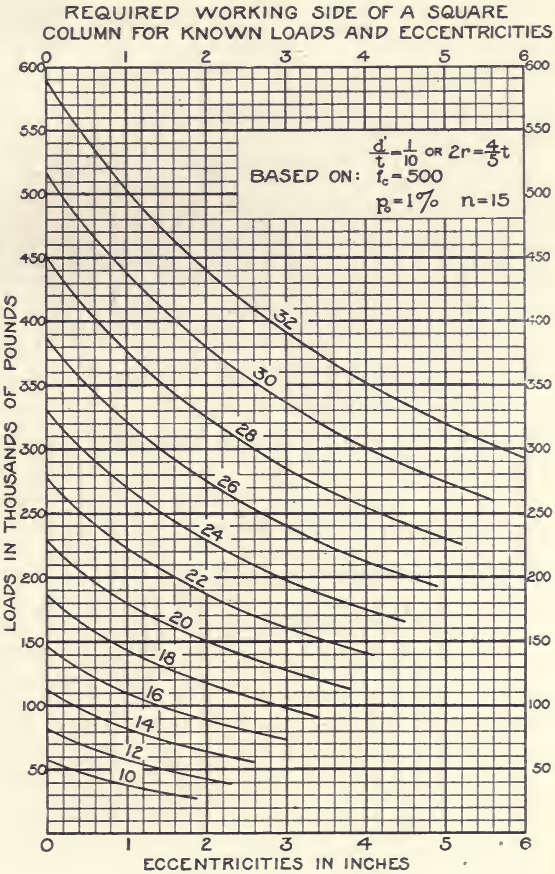

FIGURE 8

DESIGN OF INTERIOR COLUMN

Weight of roof supported by one column . . . . 38970 pounds

Weight of floor supported by one column. . . . . 67070 pounds

Weight of three floors and roof supported by one column . . . . .240180 pounds

Live load on one floor beam . . . . 18000 pounds

The moment produced in the girder due to this live load on the floor beam is 648000 inch pounds, and one-fourth of this moment is 162000 inch pounds. The approximate value of x $= \frac{M}{W} =$

$\frac{162000}{240180}$ = .67 inches, and referring to Figure 8 the probable size of column is found to be 23″ x 23″. With one and one-half inches of fireproofing on all sides the outside dimensions of the column are 26″ x 26″. The weight of one column is $\frac{26 \times 26}{144} \times 150 \times 12 = 8450$ pounds, and of three columns 25350 pounds.

The total load to be supported by the basement column with the main floor loaded on one side of the column only is 240180 + 25350 — 18000 = 247530 pounds. $x_o = .65$ and referring again to Figure 8, the column selected is of sufficient size.

As it is uneconomical to use steel reinforcement in columns, the smallest amount allowable by the specifications is used. One per cent. of 23 x 23 = 5.29 square inches, which can be furnished by twelve $\frac{3}{4}$ inch round rods, the combined area of which is 5.30 square inches.

$$a + na_s = 23 \times 23 + 15 \times 5.30 = \dots\dots \quad 609$$

$$r = \frac{4 \times 10\frac{1}{2} + 2 \times 3\frac{1}{2}}{6} = \dots\dots \quad 8.17$$

$$I_c = \frac{(23)^4}{12} = \dots\dots \quad 23320$$

$$I_s = 5.30 \times (8.17)^2 = \dots\dots \quad 354$$

$$I = 23320 + 15 \times 354 = \dots\dots \quad 28630$$

$$f_c = \frac{247530}{609} + \frac{162000 \times 11.5}{28630} = 407 + 65 = \dots\dots \quad 472$$

And checking this result by another method

$$p_o = \frac{12 \times .4418}{23 \times 23} = \dots\dots \quad .0100$$

$$\frac{r}{t} = \frac{8.17}{23} = \dots\dots \quad .355$$

$$\frac{r^2}{t^2} = \dots\dots \quad .126$$

$$z = .126 \times 180 = \dots\dots \quad 22.7$$

$$K = \frac{1}{1 + 15 \times .0100} + \frac{.65}{23} \times \frac{6}{1 + 22.7 \times .0100} = .870 + .138 = 1.008$$

$$f = \frac{247530}{23 \times 23} \times 1.008 = \dots\dots \quad 472$$

Using Diagram 13, K = 1.00 and f_c = 468.

Under full load, with no eccentricity to consider, the stress in the concrete only reaches 436 pounds per square inch. According to the specifications the rods in the column are held together by one-quarter inch bands placed about 11½ inches center to center.

DESIGN OF EXTERIOR COLUMN.

The weight of roof supported by one column is:

Live load 180 square feet at 35 pounds	6300 pounds
Slab load 180 square feet at 50 pounds	9000 pounds
Weight of 1½ beams below the slab 1½ x 1650	2470 pounds
Weight of ½ girder below the slab ½ x 5070	2540 pounds
	20310 pounds

In a similar manner the weight from one floor is 34950 pounds.

Assuming that a six inch wall is to fill the space between columns and beams, the weight on each beam due to the wall above it is $(12 - \frac{22}{12})(20 - \frac{26}{12})(\frac{6}{12})$ 150 = 13610 pounds. As this is less than one-half of the live and slab loads carried by one floor beam (28080 pounds) if the wall beam is made equal in section to the intermediate beam it will sustain the wall in addition to its proportion of floor load. The weight of walls, floors, and roof is then 165990 pounds, and the total load on one floor beam being 29830 pounds, the eccentric moment is 1,073,880 inch pounds and the approximate value of x_o is 1.62 inches. From Figure 8 the probable size of the column is 22″ x 22″ and with 1½ inches of fireproofing on all sides the outside dimensions of the column are 25″ x 25″, and the weight of one column 7810 pounds. The total load is 189420 pounds, and the actual value of x_o is 1.42 inches, and referring again to Figure 8 the size of the column selected is sufficient. One per cent. of 22 x22 = 4.84 square inches, and as eight ⅞ inch round rods furnish only 4.81 square inches, twleve ¾ inch round rods furnish the smallest section allowed by the specifications.

$a_c + na_s = 564$, $I_c = 19520$, $r = 7.78$, $I_s = 321$, $I = 24330$, and $f_c = 458$.

$p_o = .0109$, $\frac{r}{t} = .353$, $\frac{r^2}{t^2} = .125$, $z = 22.5$, $K = 1.170$, and $f_c = 458$.

Considering the slight difference in the size of the interior and exterior column it would be advisable to make them of the same size for the sake of the uniformity of construction.

COLUMN FOOTINGS.

In the design under consideration it is assumed that no property line interefers with the construction of the footings and a simple square footing is to be designed for each column.

Recent tests have shown that it makes little or no difference whether rods in a footing are placed in two or in four directions. The design of the two way type follows. For an example of the four way footing see *Concrete Plain and Reinforced* by Taylor and Thompson or *Reinforced Concrete Construction* Vol II by George A. Hool.

Tests have also shown that a diagonal tension failure will not occur nearer the edge of the column than at a distance equal to the depth of the footing at the edge of the column. In most footings where this condition is considered no web reinforcement is required and in case the unit shear in the concrete does slightly exceed the allowable value at the section mentioned above, a slight increase in the depth of the footing will bring the unit shear within the limiting value. For a design of a footing with stirrups see *Concrete Plain and Reinforced.*

DESIGN OF INTERIOR COLUMN FOOTING

Load from Roof .... 38970 pounds

Load from three floors .... 201210 pounds

Load from four columns .... 33800 pounds

Bearing area required $\frac{273980}{4000}$ .... 68.50 sq. ft.

Allowing about 10% for the weight of the footing and taking the length of the side of the footing to the nearest one-half foot, a footing 9′ - 0″ x 9′ - 0″ furnishing an area of 81 square feet is adopted.

Referring to Figure 9 the area of the trapezoid *abcd* is $\dfrac{81 - \dfrac{23 \times 23}{144}}{4}$ $= 19.33$ square feet. The upward pressure of the soil on this area is $19.33 \times 4000 = 77300$ pounds. The distance of the center of gravity of the trapezoid from the edge of the column (*ab*) is

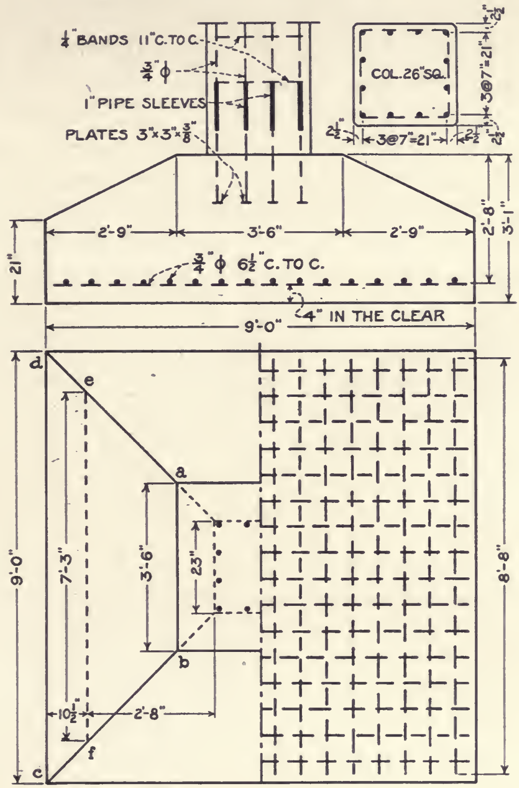

FIGURE 9

$$\frac{42\frac{1}{2}}{3}\left(\frac{23 + 2 \times 108}{23 + 108}\right) = 25.85$$ inches, and the bending moment at the edge of the column is 1998000 inch pounds.

Depth required for moment $\sqrt{\dfrac{1998000}{107.4 \times 23}} = 28.4$ inches

Depth required for punching shear $\frac{77300}{23 \times 105} = 32.0$ inches

$$a_s = \frac{1998000}{16000 \times .875 \times 32} = \ldots\ldots\ldots\ldots\ldots 4.46 \text{ square inches}$$

$$\Sigma_o = \frac{77300}{80 \times .875 \times 32} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 34.5 \text{ inches}$$

The effective reinforcement is to be spaced over a distance equal to the width of the column plus twice the depth of the footing at the edge of the column plus one-half of the remaining distance = $23 + 2 \times 32 + \frac{21}{2} = 97\frac{1}{2}$ inches; $\frac{3}{4}$ inch round rods spaced $6\frac{1}{2}$ inches center to center furnish in a width of $97\frac{1}{2}$ inches an area of 6.63 square inches and a total perimeter of 35.3 inches.

The top of the footing is made 42 inches square, thus providing a 6 inch ledge all around the outside of the column so that the column forms may be erected before the basement floor is poured. The depth T is taken equal to about one-half of d or 16 inches. Then the depth of the footing at a distance of 32 inches from the working side of the column (along ef) is $16 + \frac{10\frac{1}{2}}{33} \times 16 = 21.10$ inches, and the distance ef is $108 - 2 \times 10\frac{1}{2} = 87$ inches. The area of the trapezoid $cdef$ is 7.11 square feet, making the upward pressure from the soil on this area 28440 pounds and the unit shear along ef equal to $\frac{28440}{87 \times 21.1 \times .875} = 17.7$ pounds.

The total maximum depth of the footing at the column edge is made 37 inches and at the outside edge 21 inches, thus allowing 4 inches clear insulation below the lower layer of rods.

The weight of this footing is about 31000 pounds, making the total load sustained about 305000 pounds, requiring an area of 76.13 square feet, or a footing 8.72 feet square. Therefore no revision is necessary.

DESIGN OF EXTERIOR COLUMN FOOTING

Load from roof	20310 pounds
Load from three floors	104850 pounds
Load from three walls	40830 pounds
Load from four columns	31240 pounds
	197230 pounds
Bearing area required	49.31 square feet

Proceeding as in the design of the interior column footing, a footing 7′–6″ x 7′–6″ is selected. The area of the trapezoid *abcd* is 13.22 square feet, the upward pressure of the soil on this area 52900 pounds, and the bending moment at the edge of the column 934,000 inch pounds. The depth required for moment is 19.9 inches, and that for punching shear 22.9 inches. Adopting a depth of 23 inches the steel area required is 2.90 square inches and the total perimeter 32.8 inches. The effective reinforcement will be spaced over a distance of 79 inches, and $\frac{3}{4}$ inch round rods spaced $5\frac{1}{2}$ inches center to center will furnish in a width of 79 inches an area of 6.35 square inches and a total perimeter of 33.8 inches.

Making the top of the footing 42 inches square and the depth T, 12 inches, the depth of the footing at a distance of 23 inches from the working side of the column is 17.04 inches and the distance *ef* 68 inches. The area of the trapezoid *cdef* is 6.03 square feet, the upward pressure from the soil on this area 24120 pounds and the unit shear along *ef* 23.8 pounds. The total maximum depth of the footing at the column edge is made 28 inches and at the outside edge 17 inches.

FORMS.

1. The forms are to be like those shown on posted sheet, and in accordance with the following specifications:

Material to be of spruce, and the allowable unit stresses as follows

Modulus of elasticity	1300000 pounds per square inch
Extreme fibre stress in flexure .	1200 pounds per square inch
Compression on sides of fibres	300 pounds per square inch
Compression on ends of fibres .	1200 pounds per square inch
Shear parallel to the grain	80 pounds per square inch
Compression on struts of length under 15 times least diameter of side	900 pounds per square inch
Compression on struts over 15 times least dimension d. 1200 $(1-l/60d)$	pounds per square inch

Deflection not to exceed span / 360.

Basing the deflection on the values specified above with E = 1,300,000 and D = Span / 360, the following equations result and are to be used:

Simple beams. Equation 1, $W = 277{,}000\ I/l^2$

Equation 2, $l = \sqrt[3]{277{,}000\ I/w}$

Partially continuous beams.

Equation 3, $W = 462{,}000\ I/l^2$

Equation 4, $l = \sqrt[3]{462{,}000\ I/w}$

On the basis of strength

Simple beams. Equation 5, $W = \dfrac{9600 . I}{l\quad e}$

Equation 6, $l = \sqrt{\dfrac{9600 . I}{w\quad e}}$

Partially continuous beams

Equation 7, $W = \dfrac{12000 . I}{l\quad e}$

Equation 8, $l = \sqrt{\dfrac{12000 . I}{w\quad e}}$

In the above formulae W = total load in pounds on the span; w = the load per linear inch of span; l = the length of the unsupported span in inches; I = the moment of inertia of the board or plank in biquadratic inches.

The forms shall be designed for a construction live load of 75 pounds per square foot, in addition to the weight of the concrete they sustain. The weight of the forms may be neglected.

In the design of the column forms, the yokes are to be spaced on the assumption that the wet concrete exerts a fluid pressure of 80 pounds per square foot on the lagging, but the yokes in no case are to be spaced further apart than 24 inches center to center.

The thickness of lumber for the forms depends upon the number of times it is to be used and the manner in which it is erected, but the general practice is 1 inch stock dressed to $\frac{7}{8}$ inch for floors, $1\frac{1}{4}$ or $1\frac{1}{2}$ inch stock dressed to $1\frac{1}{8}$ or $1\frac{3}{8}$ for columns and the sides of beams and girders, and 2 inch stock dressed to $1\frac{3}{4}$ inches for beam and girder bottoms.

The actual dimensions of the material are to be used in designing and the various parts of the forms are to be made from the following stock:

	Actual Size	Nominal Size
Lagging	$\frac{7}{8}''$ or $1\frac{1}{8}''$ by $3\frac{3}{4}''$, $5\frac{3}{4}''$ or $7\frac{3}{4}''$	$1''$ or $1\frac{1}{4}''$ by $4''$, $6''$, or $8''$
Planks	$1\frac{3}{4}''$ by $5\frac{3}{4}''$, $7\frac{3}{4}$ or $9\frac{3}{4}$	$2''$ by $6''$, $8''$ or $10''$
Battens	$1\frac{7}{8}''$ by $3\frac{7}{8}''$ rough	$2''$ by $4''$
Joists	$1\frac{7}{8}''$ by $3\frac{3}{4}''$, $5\frac{3}{4}$, or $7\frac{3}{4}$	$2''$ by $4''$, $6''$, or $8''$
Yokes	$1\frac{7}{8}''$ by $3\frac{7}{8}''$ rough	$2''$ by $4''$
Posts	$3\frac{7}{8}''$ by $3\frac{7}{8}''$ rough	$4''$ by $4''$
Caps	$3\frac{7}{8}''$ by $3\frac{7}{8}''$ rough	$4''$ by $4''$
Braces	$\frac{7}{8}''$ by $3\frac{7}{8}''$ rough	$1''$ by $4''$
Stringers	$3\frac{7}{8}''$ by $5\frac{7}{8}''$, $5\frac{7}{8}''$ by $5\frac{7}{8}''$ or $5\frac{7}{8}''$ by $7\frac{7}{8}''$ rough	$4''$ by $6''$, $6''$ by $6''$, or $6''$ by $8''$,

DESIGN OF THE FORMS

Span of floor beams	$20'-0''$
Center to center floor beams	$9'-0''$
Span of girder	$18'-0''$
Thickness of slab	$4''$
Cross-section of beam below slab	$17\frac{1}{2}'' \times 7\frac{3}{4}''$
Cross-section of girder below slab	$29\frac{1}{4}'' \times 13\frac{1}{2}''$
Distance floor to floor	$12'-0''$
Cross-section of column	$26'' \times 26''$

Spacing of Joists. The weight of slab and construction live load is 125 pounds per square foot or 10.4 pounds per linear inch of 12 inch strip. The I of a $12'' \times \frac{7}{8}''$ section is .67 biquadratic inches and the safe span of the lagging by Equation 4 is 31.0 inches.

When the span of the joist is more than about 5 feet, $2'' \times 8''$ joists are required in order that they may be spaced far enough apart to develop a reasonable percentage of the strength of the lagging. It then becomes more economical to support the joists midway between floor beams, allowing the use of much smaller joists.

The distance between the centers of the joist bearers attached to adjacent floor beams is $108 - 7\frac{3}{4} - 2\ (1\frac{1}{8} + 1\frac{7}{8}) - 2 = 92\frac{1}{4}$ inches. Then with a center support the span of the joist is about 46 inches.

$\frac{I}{e}$ of $1\frac{7}{8}'' \times 5\frac{3}{4}''$ joist .... 10.35 cubic inches

$\frac{I}{e}$ of $1\frac{7}{8}'' \times 3\frac{3}{4}''$ joist .... 4.40 " "

By Equation 7 $W = 2700$ pounds for $1\frac{7}{8}'' \times 5\frac{3}{4}''$ joist

and 1150 pounds for $1\frac{7}{8}'' \times 3\frac{3}{4}''$ joist

The required spacing of the larger joist is $\dfrac{2700 \times 12}{125 \times \frac{46}{12}} = 67.6$ inches

and of the smaller 28.8 inches. The latter is very close to the spacing allowed by the lagging and the smaller joist is adopted.

The actual spacing of the joists may be determined as follows:

Center to center of girders	$20'-0''$
Width of girder	$13\frac{1}{2}''$
Side lagging $2 \times 1\frac{1}{8}$ =	$2\frac{1}{4}''$
Battens $2 \times 1\frac{3}{4}$	$3\frac{1}{2}''$
Joist	$1\frac{3}{4}''$
	21 inches

Center to center of joists attached to girders ...... $18'-3''$

Approximate number of spaces $\dfrac{18'-3''}{28.8}$ 7.61

Therefore, joists are spaced $\dfrac{18'-3''}{8}$ .... $27\frac{3}{8}''$ center to center.

Supports for the beam forms:

Construction live load per lineal foot of beam $\dfrac{7\frac{3}{4}}{12} \times 75 = 48$ pounds

Weight of beam per linear foot $\dfrac{150 \times 21\frac{1}{2} \times 7\frac{3}{4}}{144}$ = ... 174 pounds

222 pounds

Total load per linear inch	18.5 pounds
I of $7\frac{3}{4}'' \times 1\frac{3}{4}''$ plank	3.46 pounds
Safe span by Equation 4	44.2 inches

Posts supporting the beam form:

Construction live load per linear foor $\left(\dfrac{108-46}{12}\right) 75 = 388$ pounds

Weight of slab per linear foot $\left(\dfrac{108-46}{12}\right) 50$ = ... 258 pounds

Weight of beam below slab per linear foot

$\dfrac{17\frac{1}{2} \times 7\frac{3}{4} \times 150}{144}$ = ... 141 pounds

787 pounds

Total load per linear inch	65.6 pounds
Safe bearing of post on fibres of cap $3\frac{7}{8} \times 3\frac{7}{8} \times 300$ =	4500 pounds

Safe spacing of posts $\frac{4500}{65.6}$ = ... 68.6 inches

The posts are spaced $54\frac{3}{4}$ inches center to center, thus bringing a post under each alternate joist. Under the remaining joists a plank support for the beam form is placed.

Center support for joists (Stringer):

The load sustained by each joist over a span of 46 inches is:

Construction live load $\frac{27\frac{3}{8} \times 46}{144} \times 75$ = ... 656 pounds

Slab load $\frac{27\frac{3}{8} \times 46}{144} \times 50$ = ... 437 pounds

1093 pounds

One post under the center of the stringer would sustain the load from the joist without injuring the fibres of the stringer, but it would require a 6″ x 8″ stringer. By placing a post under each alternate joist the bending moment for the span of $54\frac{3}{4}$ inches is $\left(\frac{1093}{2} \times 27\frac{3}{8}\right)\frac{8}{10} = 11970$ inch pounds. Then $bd^2 = 59.9$ and with $b = 3\frac{7}{8}$ inches, $d = 3.93$ or $5\frac{7}{8}$ inches. By using a 6″ x 4″ stringer with three posts the amount of lumber is less than with a 6″ x 8″ stringer and only one post and the use of the heavier timber is avoided.

Supports for the girder forms. In a manner similar to that used in determining the spacing of supports for the beam forms the safe spacing is found to be 39.3 inches.

Posts supporting the girder forms. The joist adjacent to the girder is supported by the girder forms which are in turn supported by the posts.

Area of slab supported per linear foot 12″ (21″ + $27\frac{3}{8}$″)

Load from slab per linear foot $\frac{12 \times 48\frac{3}{8}}{144} \times 125$ = .. 504 pounds

Weight of girder below slab per linear foot

$\frac{29\frac{1}{4} \times 13\frac{1}{2}}{144} \times 150$ = ... 412 pounds

916 pounds

Total load per linear inch......................... 76.3 pounds

Safe spacing of posts . . . . 59.0 inches

Reaction from stringer not included in the above, 547 pounds.

The safe distance between the center and adjacent posts is

$$\frac{4500 - \frac{547}{2}}{76.3} = 55.4 \text{ inches}$$

The actual spacing of the posts may be determined as follows:

Center to center of column	18′ – 0″
Width of column	26″
Side lagging 2 x $1\frac{1}{8}$	$2\frac{1}{4}$
	$28\frac{1}{4}$
Clear distance between column forms	15′ – $7\frac{3}{4}$″

With one post in the center and one on each side 47 inches ($46\frac{15}{16}$) center to center the full distance will be covered. A plank support may be placed midway between the posts, but as the clear span between the posts is only 43 inches this is hardly necessary.

Column Forms:

I of 12″ x $1\frac{1}{8}$″ section 1.42

Pressure at bottom on strip 12″ x 1″ = $\frac{12 \times 80}{12}$ . . . . 80 pounds

Pressure at $\frac{1}{4}$ point on strip 12″ x 1″ = ... 60 pounds

Pressure at $\frac{1}{2}$ point on strip 12″ x 1″ . . . . 40 pounds

Pressure at $\frac{3}{4}$ point on strip 12″ x 1″ . . . . 20 pounds

SAFE SPAN OF LAGGING

Point	By Equation 4	By Equation 8
Bottom	20.2 inches	19.4 inches
$\frac{1}{4}$	22.2 inches	22.4 inches
$\frac{1}{2}$	25.4 inches	27.5 inches
$\frac{3}{4}$	32.0 inches	38.9 inches

The span of the 2 x 4 is about $26 + 2\,(1\frac{1}{8} + 1\frac{7}{8} + 1) = 34$ inches

I of 2 x 4 . . . . 9.1

W by Equation 5 . . . . 1325 pounds

SAFE SPACING OF YOKES

Bottom	$\frac{1325}{80 \times \frac{20}{12}}$	7.6 inches
$\frac{1}{4}$		10.2 inches
$\frac{1}{2}$		15.3 inches
$\frac{3}{4}$		30.5 inches

As the strength of the yokes governs their spacing, the bolts must take a stress of $\frac{1325}{2}$ pounds requiring a section of .0414 square inches. A $\frac{3}{8}$ inch bolt has a net area of .068 square inches and is adopted. A washer having a net area of $\frac{1325}{2 \times 300} = 2.21$ square inches and a gross area of 2.40 square inches must be used. A $1\frac{3}{4}$ inch round washer furnishes this area and a thicknes of $\frac{1}{8}$ inch is sufficient.

DESIGN OF A COMBINED FOOTING

When, on account of limited ground area it is impossible to construct simple spread footings to support the exterior columns,

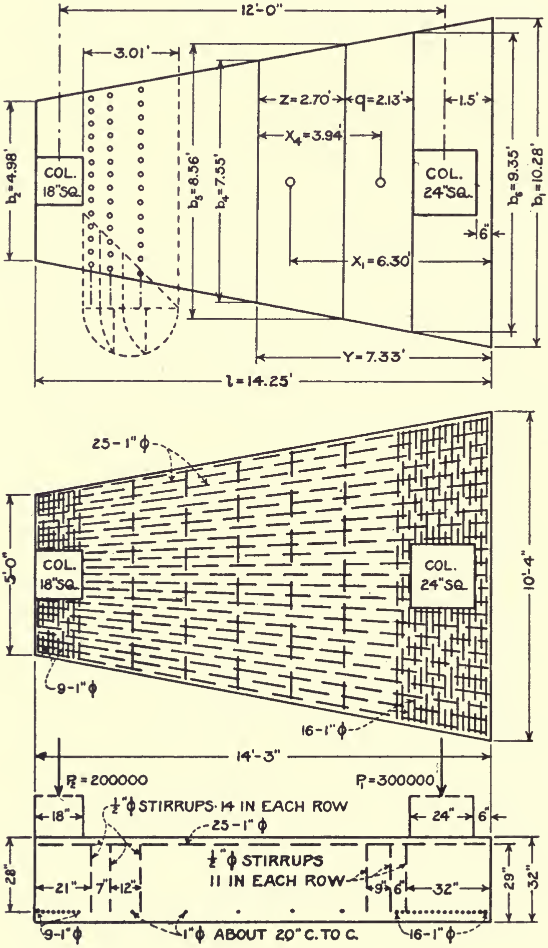

FIGURE 10

either a cantilever construction must be used or the footing must be combined with one or more of those for the interior columns.

The simplest type of a combined footing is a trapezoidal slab the center of gravity of which coincides with the center of gravity of the loads that the footing sustains. The following design shows the general manner of determining the shape of this type of footing, its depth, and the necessary amount of reinforcement.

Two columns 12′–0″ center to center, one having a cross-section of 18″ x 18″ and the other a cross-section of 24″ x 24″ sustain loads of 200000 pounds and 300000 pounds respectively. The allowable pressure on the soil is 5000 pounds per square foot, and the allowable bond stress on the reinforcing rods is 100 pounds per square inch. Otherwise the specifications heretofore used are to be followed. (See Figure 10).

The bearing area required for these loads is 100 square feet. Allowing the footing to project six inches beyond the edge of the larger column the total length of the footing is 14.25 feet.

The center of gravity of the footing must be at a distance of $\frac{200000}{500000} \times 12 + 1.5 = 6.30$ feet from b_1. Using the equations involving the area and center of gravity of a trapezoid

$$b_1 + b_2 = \frac{2 \times 100}{14.25} = 14.04$$

and

$$b_1 + 2b_2 \div \frac{3 \times 14.04 \times 6.30}{14.25} = 18.64$$

whence $b_1 = 9.44$ feet and $b_2 = 4.60$ feet. The width of the footing at a point midway between the two columns is $9.44 - \frac{7.5}{14.25}(9.44 - 4.60) = 6.94$ feet and the center of gravity of the larger of the two trapezoids on either side of this section is $\frac{7.5}{3}\left(\frac{6.94 + 2 \times 9.44}{6.94 + 9.44}\right) = 3.94$ feet from this mid-point. The bending moment at this section is $300000\,(6.0 - 3.94)12 = 7{,}310{,}000$ inch pounds and the approximate depth of footing required is 28.6 inches. Allowing for insulation, the total depth of the footing is assumed as 32 inches, which adds to the weight to be sustained 400 pounds per square foot.

The above computation was only for the purpose of determining the approximate weight of the footing. Final dimensions may now be computed in the same manner.

The area required is 108.7 square feet, and the revised values of b_1 and b_2 are 10.28 feet and 4.98 feet respectively. The section of true maximum moment is of course at the point where the shear is equal to zero. Let the distance of this point from b_1 be called y, the load on the larger column P_1, and the difference between the allowable soil pressure and the weight of the footing w. Then $P_1 = w\left(b_1y - \frac{(b_1 - b_2)\ y^2}{2\ l}\right)$, from which the distance y is determined as 7.33 feet. The width of the footing at this point (b_4) is 7.55 feet and the distance of the center of gravity of the trapezoid bounded by b_1 and b_4 is 3.85 feet from b_4.

The maximum bending moment is $300000\ (7.33 - 1.5) - 3.85 \left(\frac{7.55 \times 10.28}{2}\right) 7.33 \times 4600 = 590000$ foot pounds or 7080000 inch pounds and the depth of footing required is 27.0 inches.

The width of the transverse distributing beam under the larger column is taken as 36 inches and that under the smaller column as 18 inches. The moment at the edge of the column due to the upward pressure of the soil is $\frac{300000}{8 \times 10.28}\ (10.28 - 2.0)^2 \times 12 = 3000000$ inch pounds for the longer beam, and $\frac{200000}{8 \times 4.98}\ (4.98 - 1.5)^2 \times 12 =$ 732000 inch pounds for the shorter beam. The maximum shear at the edge of the column for the former is $\frac{300000}{2}\left(\frac{10.28 - 2.0}{10.28}\right) =$ 121000 pounds, and for the latter 70000 pounds.

The depths required are 27.9 inches and 19.5 inches respectively. A depth of 28 inches is adopted for the entire footing and with four inches of insulation the total depth is 32 inches as assumed.

The distance from the point of zero shear to the point where the unit shear reaches the allowable value for plain concrete is determined by solving the following equation for z. $z^2\left[\frac{w}{2l} - (b_1 - b_2)\right] + z\left[wb_4 - \frac{v'jd(b_1 - b_2)}{l}\right] - v'jdb_4 = 0$ (In using this equation inches must be used with pounds per square inch, and feet with pounds per square foot). From the above equation the value of z is 2.70 feet, and the width of the footing at this point (b_5) is 8.56 feet and that at the edge of the larger column (b_6) is 9.35 feet. The distance over which web reinforcement is required (q) is 2.13 feet. The maximum shear along b_6 is $\frac{7.55 + 9.35}{2}$ (4.83) (4600) = 187700 pounds, and the unit shear 68.3 pounds. The average unit shear over the distance q is 54.15 pounds, the average width of the footing 8.95 feet and the total area of web reinforcement required $\frac{2 \times 54.15 \times 8.95 \times 2.13 \times 144}{3 \times 16000} = 6.20$ square inches, requiring thirty-two ½ inch round single stirrups. The spacing of the stirrups is determined graphically on Figure 10.

In a similar manner forty-two ½ inch round single stirrups spaced over a distance of 3.01 feet are required at the other end of the footing. Computing the horizontal reinforcement for the main slab

$$a_s = \frac{7080000}{16000 \times .875 \times 28} = 18.1 \text{ square inches and}$$

$$\Sigma_o = \frac{187700}{100 \times .875 \times 28} = 76.6 \text{ inches; which will be furnished by}$$

twenty-five 1 inch round rods. Similarly for the transverse beams sixteen rods and nine rods respectively are required.

The same condition with regard to diagonal tension failures may be assumed for the distributing beams as was assumed in the square footing.

Computing the unit shear 28 inches from the edge of the column it is found to be 59.7 pounds so that stirrups are required from this point to the point where the shear is equal to 40 pounds per square inch which is at a distance of 33 inches from the edge of the column. The area of web reinforcement required is $\frac{2 \times 49.9 \times 36 \times 5}{3 \times 16000} = .374$ square inches so that one double ½ inch round stirrup will be sufficient. Two double stirrups are placed 30 inches from the edge of the column in order to distribute the reinforcement over the full width of the beam.